

## Design of $H_\infty$ for induction motor

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### ABSTRACT

For Induction motor is a system that works at their speed, nevertheless there are applications at which the speed operations are needed. The control of range of speed of induction motor techniques is available. The robust control is used with induction motor and the performance of the system with the controller will be improved. The mathematical model to the controller, which were coded in MATLAB. The modeling and controller will be shown by the conditions of robustness of  $H_\infty$  be less than one.

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### 1. INTRODUCTION

The small signal besides steady state parameters of cage induction motor are estimated in a big range operation by using finite-element (FE) method. A machine designed for a frequency converter supply [1]. On the other hand, the parameters of three-phase for induction motors can determine by implementation of a rapid online method, this procedure is built data sampling through making the speed a normal run up exam. And also the locked-rotor besides the synchronous speed data have been tested during the run up exam [2]. One of advance method for computes the required system signals are (MPC) means model predictive control, this method is consider direct and easy algorithm and for nonlinearities in the technique [3]. A sliding mode (MRAS) benefits to keep the stability in case low speed area.  $H_\infty$  method implements to calculate the sliding mode gain, that progress the robustness of the observer system compare with the speed of induction machine that independent of voltage mode. This procedure minimizes the impact of the error during the prediction and observer model [4]. Another method used Self Tuning (ST) technique depend on Takagi-Sugeno (TS) fuzzy rules [5].

The  $H_\infty$  controller consider the solution to a maximum and the minimum differential game, the controller can reduce a quadratic cost function related to the error of state vector from the machine, whereas the designing errors and external disturbance attempt to reduce it [6-10].

There are two stages method to control the induction motor, the first is by using the first order of Taylor series extension to linearize of the dynamic model motor so can use Jacobian matrices to compute it. Then the second stage used linearized model for the induction motor by solving an algebraic Riccati equation to design an  $H_\infty$  feedback control rule by calculating in each repetition of the control algorithm [11].

Still the derivative of the transfer function of the motor by manual method is useful and has an important role to simplify assumption of the deviation [12]. Recently compute the poles of transfer function has more attractive for the researchers, which can compute the poles by the eigenvalues of the structural state matrix [13]. In this paper proposed a method to design robust with I-P controller for an induction motor, and the dynamic behavior of induction motor is shown with the transfer function motor. The induction motor has problems in the control if we use many variables in designing of the system because in the control such as P, PI, PID, or MPC controller, it needs one variable as input and one output but it doesn't work with many variables.

In this paper we found a solutions to the problem of controlling with the variables of the system by mixing I-P controller with robust control ( $H_\infty$ ) to get the robustness and stability with whole of the variables and this controller is very good for designing [14]. There are many papers used a robust control for induction motor but in this paper consider the best because it used with I-P controller with whole ranges of variables [15-19]. The superconducting induction motor, which developed an analysis model with equation of a motor voltage based on the non-linear current that related to HTS windings. The higher efficiency control of the motor by usage of behavior of the aforementioned hysteretic. They got a good results in both load and no load for the hysteretic rotating characteristics and got the peak value of efficiency when the voltage is decreasing after rotating synchronous of the motor. The characteristics of high temperature super conductor induction motor has been indicated that the HTS electric power with a synchronous at the efficiency is more than 90% and rated condition 20kW. The characteristics have been tested and replicated relied on equivalent circuit of nonlinear electrical. The results was comfortable of extremely organized for performance of synchronous regeneration [20-21].

The induction motors has been used for conversion between mechanical and electrical (electromechanical) and are show in most processes of production for two thirds of the consumption of industrial electric. Induction motors faults can display in human losses, stop entire sectors of a plant, operational disasters and causing economic. The techniques are critical for diagnosis of fault in induction motors. A hybrid system that uses data got from current sensors and vibration to discover failures at early step. The failures were correctly due to the load with unbalanced way in the motor helix and in the motor shaft [22]. The structure of cryogenic induction motor went under water with natural gas for operation LNG spray pump. The induction motor torque of induction motor dissimilar from the room temperature conditions to design specifications of the induction motor in environment in cryogenic manner. The design of cryogenic related to resistivity variation for rotor bars and stator windings [23]. The technique of direct torque control (DTC) for a two level inverter gave five phase induction motor (FPIM) for operation in low speed demands the harmonic voltage elimination would generate a current of distorted stator. The analysis with a theoretical manner is implemented to discover results of virtual vectors on flux response of FPIM and torque with speed varies to help in choosing flux band width, VVs, efficient formation of sectors and hysteresis torque band width [24].

The faults of bearing are the main root for the failures of induction motor. The methods of diagnosis of fault has been examined on tests of labs which inflexible and costly. The three – phase squirrel cage induction motor is simulated by modeling of multiple coupled circuits. The analysis is done by the effecting of saturation of the motor. The results of experimental agree with the results of simulation [25].

## 2. MODELING OF THE SYSTEM

There are three phase machines for speed of the stator by  $N_s = \frac{120f}{P}$

Where  $P$  is the number of poles and  $f$  is the frequency in  $H_z$ . Figure 1 describes the Pre-phase circuit related to the stator.

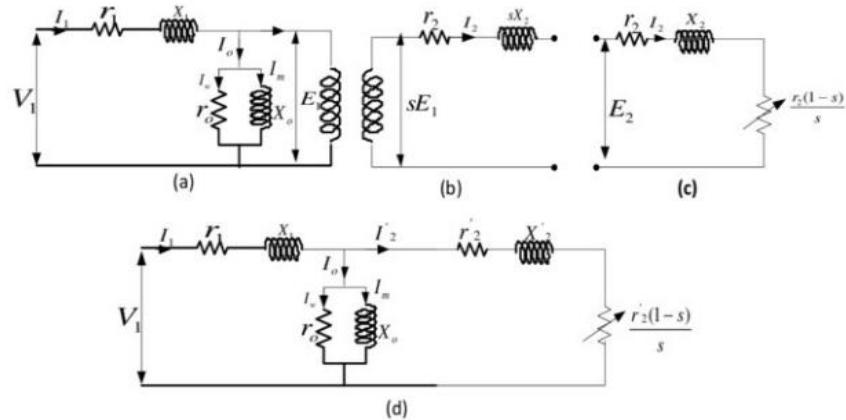


Figure 1. Phase circuit of stator side

The equation of electrical machine as shown in Figure (2) is

$$E_b = K \frac{d\theta}{dt} \dots \dots \dots \dots \quad (2)$$

The equation of electrical torque is

The equation of mechanical torque is

By taking Laplace transform for equation (1) and get

But we have from equation (2)

Substituting equation (6) in (5), we get

By taking Laplace transform for equation (4) and get

The electrical torque is

The electrical torque is equal to the mechanical torque

The transfer function is

We have

Substituting equation (14) in (13) , we will get

$$\frac{\theta(s)}{V_a(s)} = \frac{K_a}{JL_a s^3 + (R_a J + L_a B) s^2 + (BR_a + K_a K_t) s} \dots \dots \dots \quad \dots (15)$$

With parameters  $K_a = 0.019$ ,  $K_t = 0.5$ ,  $R = 4.2 \Omega$ ,  $B = 8$  and  $L = 3 \text{ mH}$

The find transfer function is

$$\frac{\theta(s)}{V_a(s)} = \frac{0.000128 s}{0.00067 s^3 + 0.7195 s^2 + s}$$

The response of output as shown in Figure 3.

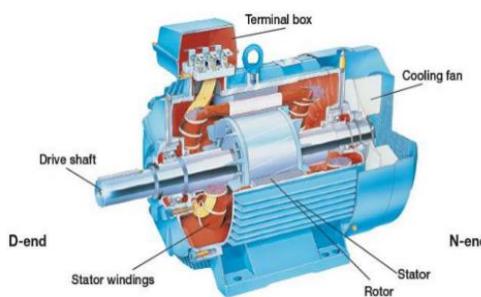


Figure 2. Induction motor

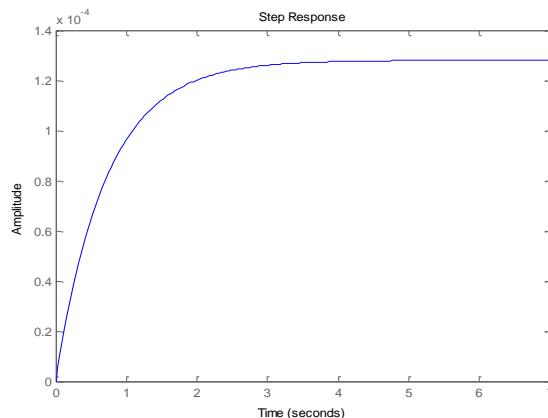


Figure 3. The output for step input

### 3. IP CONTROLLER DESIGN

The IP feedback controller for the closed-loop system with is shown in Figure 4. The transfer function with closed-loop system. Integral proportional controller (I-P) is advance form of proportional integral controller. In this method of controller the integral part is in feedforward path and proportional part is in feedback path. The disadvantage in P-I controller is that high maximum of peak overshoot. To decease that maximum of peak overshoot we can use this I-P controller. We can drive the control low of I-P controller as.

$$\begin{aligned}
 E(s) &= R(s)(s) \\
 u(s) &= \frac{K_i}{s} E(s) - K_p y(s) \\
 u(s) &= \frac{K_i}{s} R(s) - \frac{K_i}{s} y(s) - K_p y(s) \\
 u(s) &= \frac{K_i}{s} R(s) - y(s) \left[ \frac{K_i}{s} - K_p \right]
 \end{aligned}$$

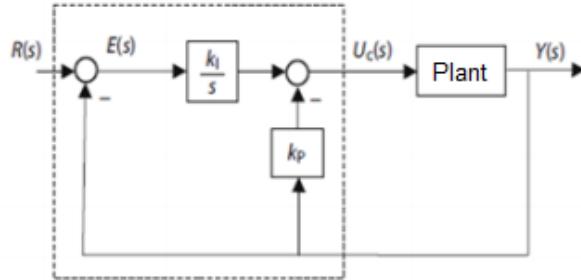


Figure 4. I-P controller diagram

#### 4. The $H_\infty$ CONTROL PROBLEM

For the purpose of selection in this paper,  $H_\infty$  methods will be required. This gets view is defined by the identification that  $H_2$  and  $H_\infty$  methodologies are alike in that: both are in need of the results to two Riccati equations, both play as controllers of state-dimension equal to the generalized plant, P, and both give an idea to structure in their controllers that are already seen in LQG control [12]. It is significant to know that  $H_\infty$  controllers give a sub-optimal controller, which contrasts with  $H_2$  control that gives identical and special controllers. It must be noted that (s) is often dropped as a usage.

The formulation of the general problem of  $H_\infty$  problems is illustrated by

$$\begin{bmatrix} z \\ v \end{bmatrix} = P(s) \begin{bmatrix} w \\ u \end{bmatrix} = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix} \dots \dots \dots \quad \dots \dots \dots \quad (16)$$

$$u = K(s)v$$

The generalized plant P has shown by

$$P = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix} \dots \dots \dots \quad (17).$$

The parameters v, the measured variables, u, the control variables, z the error signal to be minimized, w, the signal of exogenous such as disturbances. The schematic of the generalized plant model can be shown in Figure 5.

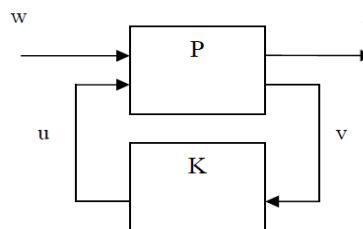


Figure 5. The Generalized plant model

By mentioning to Skogestad and Postlethwaite [2], the linear transfer functional transformation from w to z through the closed-loop transfer function will be as

$$z = F_l(P, K)w$$

Where

$$F_l(P, K) = P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21} \dots \dots \dots \quad (18)$$

## 5. MIXED SENSITIVITY $H_\infty$ CONTROL with I-P CONTROLLER

Mixed sensitivity (MS) control is the transfer function to find a controller that gives the necessary closed-loop sensitivity transfer functions  $T$ ,  $S$  and  $KS$ .  $T$  is the transfer function closed-loop which is calculated from:

$S$  is the sensitivity function which is calculated from:

These quantities (19) and (20) are the feedback configuration shown in Figure 6.

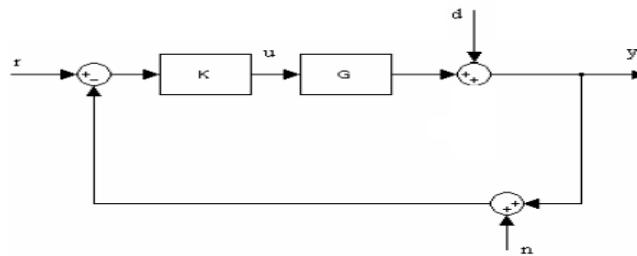


Figure 6. Feedback configuration of one degree freedom.

From Figure 3, can obtain  $y(s)$  and  $u(s)$  can be seen:

$$y(s) = S(s)d(s) + T(s)[r(s) - n(s)] \dots \dots \dots \quad .. (21)$$

These two equations calculate the closed-loop in addition to the requirement that  $K$  stabilizes to reject disturbances to the system, and the maximum singular value of  $S$  should be small to reject noise. For reduction control energy that makes the maximum singular value of  $KS$  small.

The robust stability with additional perturbations, make the value of maximum singular KS small from aims of the performance, it can be shown that the great deal of trade-off between competing objectives.

## 6. H-INFINITY SYNTHESIS

The controller  $K$  is to be tuned such that the H-infinity between the outputs and inputs and the system in close loop is stable of the matrix are less than one. If this condition is agreed, then the output of the controller is said to have a robust performance system, which means that for the range of uncertain model parameters the control of close loop meets the required performance description. The  $\mu$ -synthesis process to obtain the real controllers was done by using the `hinfssyn.m` instruction in MATLAB®. This instruction utilities a mixed sensitivity technique, which decreases the cost function containing the weights of three performances from above and this cost function should be less than one to meet performance specifications as shown in equation (23).

Where  $S$  is the sensitivity transfer function if the system is closed loop, and  $T$  is the complementary sensitivity function.

The generalized plant transfer function matrix,  $P$ , that symbolizes the system in Figure 6 and is used for the controller combination process is seen in Equation (24). Once this combination process was realized for each of controllers verification of their performance is realized to confirm the performance specifications were matched.

In Figure 7,  $W_1$  and  $W_2$  are  $w_p$  and  $w_u$  respectively. P is defined as

It is significant to note that all weights,  $w_i$ , should be stable [2].

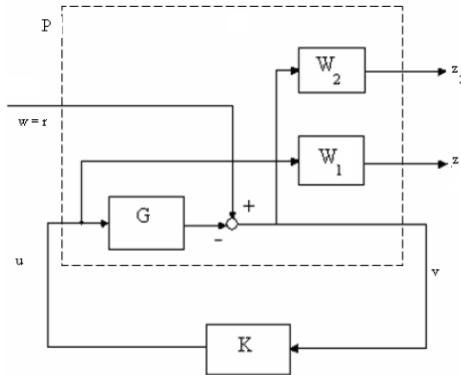


Figure 7. S/KS from of MS for tracking

## 7. PERFORMANCE ANALYSIS

Confirmation was done to make certain that each controller,  $K$ , discovered by the  $\mu$ -synthesis approach agrees robust stability requirements and the performance specifications. The controllers are nominally stable with no plant uncertainty and there should be no poles on the right-half plane for the nominal closed loop plant. The performance check was to make certain the value of (23) was less than one. Another approach includes analyzing the  $N$  matrix. The lower fractional transformation of the  $P$  matrix detailed in (24) and the controller.

Nominal performance checks that the nominal plant is controlled relating to the performance specifications.

Robust stability and robust performance to make certain that the system is stable in closed loop and agrees the given performance requirements over the given range of uncertain parameters.

The final checking is robust performance, which makes certain performance specifications are agreed for all uncertain plants.

The criteria of performance used to describe the performance, weights in some cases were different than the performance, weights used to determine the performance checks. Where  $G_p$  is the perturbation plant from the nominal and  $G_{nom}$  is the nominal plant. The nominal plant had a system gain of 2 and a time constant of 0.35 seconds. The  $G_{nom}$  was determined by gain uncertainty multiplied the nominal plant. The line solid plotted in Fig 9 is the  $W_1$  uncertainty weight bounding the maximum error for all values of frequencies by plotting the multiplicative uncertainty transfer function, and we can see that in Figures 8 and 9. The system is nominally stable if only if the values of  $N_{22}$  less than one and can be seen the nominal performance by plotting the H-infinity norm as shown in Figure 10. For the robust stability,  $N_{11}$  is less or

equal one for all frequencies and that the system is robust stable. The h-infinity norm of  $N_{11}$  can be seen in Figure 11. The  $\mu$ -synthesis controller is found to have robust performance which can be seen in Figure 12

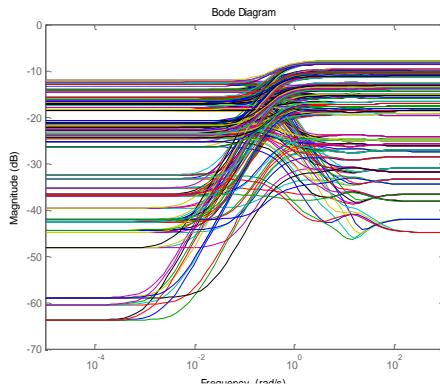


Figure 8. Transfer function bounding of Multiplicative uncertainty the maximum error for the set parameter range.

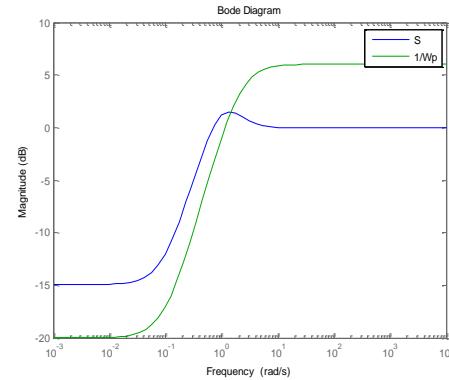


Figure 9. Bode diagram of the  $w_P$  performance weight.

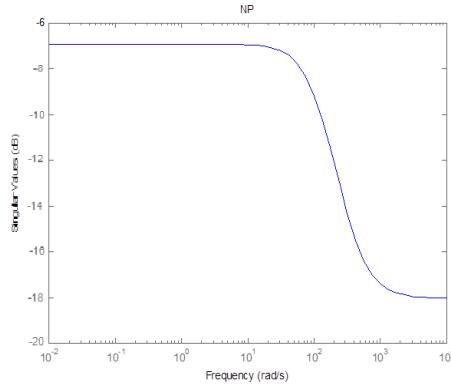


Figure 10. The  $N_{22}$  of H-infinity of is less than one for all frequencies.

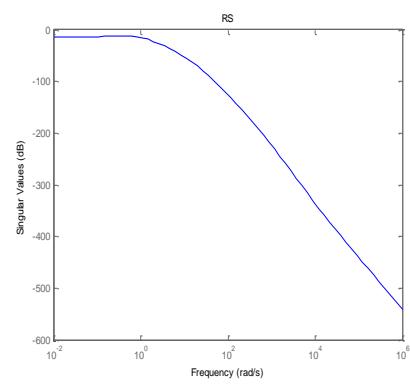


Figure 11. The  $N_{11}$  of H-infinity norm is less than one for all frequencies for the  $\mu$ -synthesis controller.

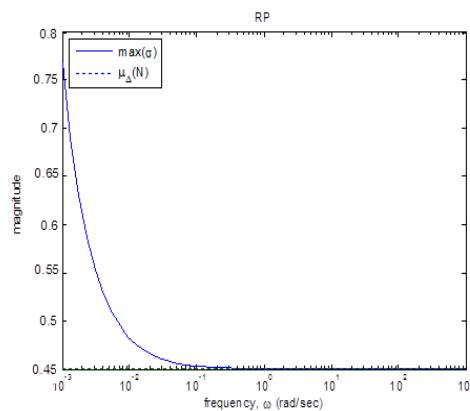


Figure 12. Structured singular value and the maximum singular value and of the N matrix is less than one for all frequencies

## 8. CONCLUSION

This paper showed a method to design a robust controllers for an induction motor. This paper has shown, from the results in time and frequency domain, which the robust control method with I-P controller can be successfully of induction motor. The dynamic behavior of induction motor is shown with the transfer

function motor is shown with the transfer function. The theory behind the robust control with the applying of conditions are very comfortable for each step of robust control to control the performance and robustness to analyze different parameters of the system. The condition of stability for performance, nominal and robustness are very comfortable and all they are less than one and is comfortable with many variables of parametrs of the system with I-P controller.

## REFERENCES

- [1] A.-K. Repo, et al., "Parameter estimation for induction motors to study the effects of voltage, frequency and slip," *Power Electronics and Applications, 2007 European Conference on*, pp. 1-10, 2007.
- [2] C. Grantham and D. J. McKinnon, "Rapid parameter determination for induction motor analysis and control," *IEEE Transactions on Industry Applications*, vol. 39, pp. 1014-1020, 2003.
- [3] J. Rodriguez, et al., "State of the art of finite control set model predictive control in power electronics," *IEEE Transactions on Industrial Informatics*, vol. 9, pp. 1003-1016, 2013.
- [4] F. Wang, et al., "Finite control set model predictive torque control of induction machine with a robust adaptive observer," *IEEE Transactions on Industrial Electronics*, vol. 64, pp. 26-2641-31, 2017.
- [5] N. Farah, et al., "Self-tuning Fuzzy Logic Controller Based on Takagi-Sugeno Applied to Induction Motor Drives," *International Journal of Power Electronics and Drive System (IJPEDS)*, Vol. 9, No. 4, pp. 1967-1975, 2018.
- [6] G. J. Toussaint, et al., " $H_\infty$  optimal tracking control techniques for nonlinear underactuated systems," in *Decision and Control, 2000. Proceedings of the 39th IEEE Conference on*, pp. 2078-2083, 2000.
- [7] L. Lublin and M. Athans, "An experimental comparison of  $H_2$  and  $H_\infty$  designs for an interferometer testbed," in *Feedback Control, Nonlinear Systems, and Complexity*, ed: Springer, 1995, pp. 150-172.
- [8] J. Dyle, "State-space solutions to standard  $H_2$  and  $H_\infty$  control problems," *IEEE Trans, on Automat Contrm*, vol. 34, pp. 831-847, 1989.
- [9] G. Rigatos, et al., "Nonlinear optimal control for Synchronous Reluctance Machines," in *2017 11th IEEE International Conference on Compatibility, Power Electronics and Power Engineering (CPE-POWERENG)*, pp. 594-599, 2017.
- [10] M. Nadour, et al., "Robust coordinated control using backstepping of flywheel energy storage system and DFIG for power smoothing in wind power plants," *International Journal of Power Electronics and Drive System (IJPEDS)*, Vol. 10, No. 2, June 2019, pp. 1110-1122.
- [11] G. Rigatos, et al., "Nonlinear  $H_\infty$  feedback control for asynchronous motors of electric trains," *Intelligent Industrial Systems*, vol. 1, pp. 85-98, 2015.
- [12] T. Lipo and A. Plunkett, "A novel approach to induction motor transfer functions," *IEEE Transactions on Power Apparatus and Systems*, pp. 1410-1418, 1974.
- [13] G. Rogers, "Linearised analysis of induction-motor transients," in *Proceedings of the Institution of Electrical Engineers*, pp. 1917-1926, 1965.
- [14] L. Alkurawy, "Design of an Efficient Controller for Arterial Oxygen Saturation in Neonatal Infants ", 2013
- [15] J. Wang, F. Wang, Z. Zhang, S. Li, and J. Rodriguez, "Design and implementation of disturbance compensation-based enhanced robust finite control set predictive torque control for induction motor systems," *IEEE Transactions on Industrial Informatics*, vol. 13, pp. 2645-2656, 2017.
- [16] F. Alonge, M. Cirrincione, F. D'Ippolito, M. Pucci, and A. Sferlazza, "Robust active disturbance rejection control of induction motor systems based on additional sliding-mode component," *IEEE Transactions on Industrial Electronics*, vol. 64, pp. 5608-5621, 2017.
- [17] S. A. Davari, F. Wang, and R. M. Kennel, "Robust deadbeat control of an induction motor by stable MRAS speed and stator estimation," *IEEE Transactions on Industrial Informatics*, vol. 14, pp. 200-209, 2017.
- [18] B. Wang, X. Chen, Y. Yu, C. Wang, B. Li, and D. Xu, "Robust predictive current control for induction motor in synchronous rotating frame," 2016.
- [19] H. Liu, Z. Mao, B. Jiang, and K. Zhang, "Robust fault-tolerant control design for induction motor with faults and disturbances," in *2016 35th Chinese Control Conference (CCC)*, pp. 6795-6800, 2016.
- [20] Kenichi Ikeda, Taketsune Nakamura, Tomoharu Karashima, Naoyuki Amemiya, Masaaki Yoshikawa, Yoshitaka Itoh, Toshihisa Terazawa, and Yoshimasa Ohashi, "Hysteretic Rotating Characteristics of HTS Induction/Synchronous Motor," *IEEE Transactions on Applied Superconductivity*, vol. 27, Issue 4, 2017
- [21] Tomoharu Karashima, Taketsune Nakamura, Kenichi Ikeda, Naoyuki Amemiya, Masaaki Yoshikawa, Yoshitaka Itoh, "Experimental and Analytical Studied on Highly Efficient Regenerative Characteristics of a 20-kW Class HTS Induction/ Synchronous Motor", *IEEE Transactions on applied superconductivity*, vol. 27, Issue 4, 2017.
- [22] G. A. Cruz, R. D. Gomes, F. A. Belo and A. C. Lima Filho, "A Hybrid System Based on Fuzzy Logic to Failure Diagnosis in Induction Motors," *IEEE Latin America Transactions*, vol. 15, Issue 8, pp. 1480-1489, 2017.
- [23] Hui Min Kim, Ki Wook Lee, Do Gyun Kim, Jong Hoon Park, and Gwan Soo Park, " Design of Cryogenic Induction Motor Submerged in Liquefied Natural Gas," *IEEE Transactions on Magnetics*, vol. 54, Issue 3, 2018.
- [24] Saifullah Payami and Ranjan K. Behera, "An Improved DTC Technique for Low Speed Operation of a Five-Phase Induction Motor", *IEEE Transaction on Industrial Electronics*, pp. 3513-3523, vol. 64, Issue 5, 2017.
- [25] Mansour Ojaghi, Mahdi Sabouri and Jawad Faiz, "Analytic Model for Induction Motors under Localized Bearing Faults," *IEEE Transaction on Energy Conversion*, vol.33, Issue 2, pp. 617-626, 2018.

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